

Hilbert space decomposition for Coulomb blockade in Fabry–Pérot interferometers

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Abstract We show how to construct the thermodynamic grand potential of a droplet of incompressible fractional quantum Hall liquid, formed inside of an electronic Fabry–Pérot interferometer, in terms of the conformal field theory disk partition function for the edge states in presence of Aharonov-Bohm flux. To this end we analyze in detail the algebraic structure of the edge states’ Hilbert space and identify the effect of the variation of the flux. This allows us to compute, in the linear response approximation, all thermodynamic properties of the conductance in the regime when the Coulomb blockade is softly lifted by the change of the magnetic flux due to the weak coupling between the droplet and the two quantum point contacts.

1 The FQHE Fabry–Pérot interferometer

The electronic version [1] of the famous optical Fabry–Pérot interferometer, which we will analyze here, is constructed by two quantum point contacts (QPC) inside of an incompressible fractional quantum Hall (FQH) bar [2, 3, 4]. In the weak-backscattering regime, small gate voltages on the QPCs create constrictions inside the incompressible FQH liquid and facilitate tunneling of non-Abelian quasiparticles along the QPCs. However, this regime is unstable in the sense of the renormalization group flow, i.e., even a small number of quasiparticles tunneling along the QPCs at low T significantly renormalizes the tunneling amplitudes thus intensifying tunneling and eventually the two QPCs pinch off, which corresponds to the strong backscattering regime that is already a stable fixed point of the renormalization group flow.

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In the strong backscattering regime, when the QPCs gate voltages are big enough that the two constrictions are completely pinched-off, the two-dimensional electron gas is split into three disconnected FQH liquids forming a *Coulomb blockade* (CB) island in the middle, see Fig. 1. Only electrons could tunnel between the disconnected parts of the interferometer and the main mechanism at low temperature and low bias is through *single electron tunneling*. The conductance in the CB regime

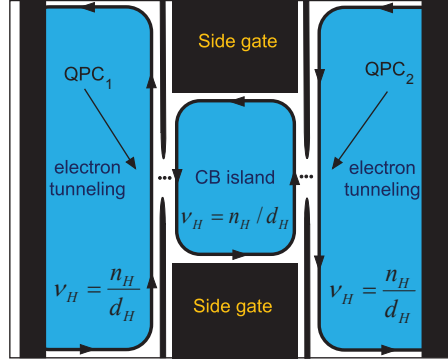


Fig. 1 A FQH bar with two QPCs in the strong backscattering regime in which single electrons could tunnel, if small bias is present, between the three disconnected liquids producing discrete peaks in the conductance. The side gates' voltage could change the area of the CB island varying in this way the flux through the island.

is determined in the following steps (sequential tunneling through CB island): first one electron tunnels from the left FQH liquid through the left QPC to the island, then the electron which is accommodated at the edge of the CB island is transported along the edge and then it tunnels through the right QCP to the right FQH liquid. Using the Landauer formula one can see [5] that the CB conductance is

$$G_{CB}(T, \phi) = \left(\frac{h}{e^2} \right) \frac{G_L G_R}{G_L + G_R} G_{is}(T, \phi), \quad (1)$$

where the CB island's conductance G_{is} depends on the magnetic flux $\phi = BA$: for most values of the flux we have Coulomb blockade ($G = 0$) and for special discrete values of flux we have conductance peaks [6, 7, 8, 9, 10]. The tunneling conductances of the two QPCs are independent of the flux and vanishing at low-temperature as $G_{L,R} \propto T^{4\Delta-2}$ where Δ is the scaling dimension of the electron operator.

2 Coulomb blockade island's conductance—the CFT point of view

An interesting observation in this setup is that the conductance of the CB island can be explicitly computed at finite temperature within the framework of the conformal

field theory [5]. This is due to the Einstein's relation [11, 5], which expresses the conductance $\sigma(0)$ in terms of the charge stiffness (or, thermodynamic density of states)

$$\sigma(0) = e^2 D \left. \frac{\partial n}{\partial \mu} \right|_T, \quad (2)$$

where D is the diffusion coefficient, μ is the chemical potential, n is the electron density and the thermodynamic derivative is at constant temperature.

The diffusion coefficient is usually related to the relaxation time [11], for normal conductors, however, for a *ballistic one-dimensional channel*, such as the FQH edge, the relaxation time must be replaced by the *time-of-flight* τ_f and the diffusion coefficient could be written as follows [11]

$$D_{\text{bal}} = v_F^2 \tau_f, \quad \tau_f = L/(2v_F) \Rightarrow D = Lv_F/2,$$

where v_F is the Fermi velocity at the edge and L is the circumference or length of the edge. According to Eq. (2) the charge stiffness can be computed as a derivative of the thermodynamic average of the particle number. To this end we shall use the Grand canonical partition function for a disk-shaped CB island derived within the CFT framework [12, 5]

$$Z_{\text{disk}}(\tau, \zeta) = \text{tr}_{\mathcal{H}_{\text{edge}}} e^{-\beta(H-\mu N)} = \text{tr}_{\mathcal{H}_{\text{edge}}} e^{2\pi i \tau (L_0 - c/24)} e^{2\pi i \zeta Q}, \quad (3)$$

where the Hamiltonian of the disk $H = \hbar \frac{2\pi v_F}{L} (L_0 - \frac{c}{24})$ is related to the zero mode of the Virasoro stress-tensor, c is the Virasoro central charge [12], v_F is the Fermi velocity of the edge states and L is the circumference of the disk; the particle number $Q \equiv N = \sqrt{v_H} J_0$ is proportional to the zero mode of the $\widehat{u}(1)$ current and v_H is the FQH filling factor.

The Hilbert space $\mathcal{H}_{\text{edge}}$ for the edge-states depends on the number and type of the residual quasiparticles which might be localized in the bulk when the magnetic field varies slightly around the value corresponding to the plateau of the Hall conductance. The thermodynamic parameters, such as the temperature and the chemical potential are related to the *modular parameters* τ and ζ on the torus introduced in a standard way for the rational CFTs [12]

$$\tau = i\pi \frac{T_0}{T}, \quad T_0 = \frac{\hbar v_F}{\pi k_B L}, \quad \zeta = i \frac{1}{2\pi k_B T} \mu. \quad (4)$$

2.1 CFT disk partition function in presence of AB flux

When magnetic field threading the CB disk or the area of the disk are changed the effect on the one-dimensional edge state's system¹ is through the variation of

¹ due to the incompressibility of the FQH droplet, the states in the bulk are localized and the only states capable of carrying electric current are living on the edge which is a one-dimensional channel

the Aharonov–Bohm (AB) flux. As can be seen in Ref. [13] introducing AB flux changes the boundary conditions of the electron field operator and naturally twists the $\widehat{u(1)}$ current and the Virasoro stress tensor. The ultimate effect of this twisting on the partition function is that it simply shifts the modular parameter as follows $\zeta \rightarrow \zeta + \phi \tau$, i.e. the partition function in presence of AB flux ϕ is

$$Z_{\text{disk}}^{\phi}(\tau, \zeta) = Z_{\text{disk}}(\tau, \zeta + \phi \tau). \quad (5)$$

The Grand potential on the edge [14]

$$\Omega(T, \mu) = -k_B T \ln Z_{\text{disk}}(\tau, \zeta) \quad (6)$$

can be used to compute the particle density in the usual way

$$\langle n \rangle_{\beta, \mu} = -\frac{k_B T}{L} \frac{\partial}{\partial \mu} \ln Z_{\text{disk}}(\tau, \zeta) = \frac{1}{L} \langle J_0 \rangle_{\beta, \mu} \quad (7)$$

where $\beta = (k_B T)^{-1}$ is the inverse temperature and the thermal average is as usual

$$\langle A \rangle_{\beta, \mu} = Z_{\text{disk}}^{-1}(\tau, \zeta) \text{tr}_{\mathcal{H}_{\text{edge}}} A e^{2\pi i \tau (L_0 - c/24)} e^{2\pi i \zeta J_0}. \quad (8)$$

2.2 Coulomb island's conductance

In order to obtain the charge stiffness of the CB island, we need to differentiate the particle density which, according to Eqs. (3) and (8), is related to the thermodynamic averages of the zero mode of the $\widehat{u(1)}$ current

$$\left\langle \frac{\partial n}{\partial \mu} \right\rangle_{\beta, \mu} = \frac{1}{L k_B T} (\langle J_0^2 \rangle_{\beta, \mu} - (\langle J_0 \rangle_{\beta, \mu})^2). \quad (9)$$

On the other hand, the Grand potential on the edge $\Omega(T, \mu)$ depends on the AB flux ϕ threading the edge because of Eqs. (4) and (5) and the second derivative with respect to ϕ is

$$\frac{\partial^2 \Omega}{\partial \phi^2} = -\frac{(h v_F / L)^2}{k_B T} (\langle J_0^2 \rangle_{\beta, \mu} - (\langle J_0 \rangle_{\beta, \mu})^2). \quad (10)$$

Comparing Eq. (9) with Eq. (10) we conclude [5] that *edge conductance is exactly proportional to the magnetic susceptibility* $\kappa(T, \phi) = -(e/h)^2 \partial^2 \Omega(T, \phi) / \partial \phi^2$, i.e.

$$G_{\text{is}}(T, \phi) = \frac{\sigma_{\text{is}}(0)}{L} = -\frac{L}{2v_F} \left(\frac{e}{h}\right)^2 \frac{\partial^2 \Omega(T, \phi)}{\partial \phi^2} \quad (11)$$

This beautiful result, which relates a non-equilibrium quantity, such as the CB islands' conductance G_{is} , to an equilibrium one expressed as a derivative of the Grand potential Ω , is valid within the Kubo linear response regime, characterized by the

conditions $G_{L,R} \ll e^2/h$, which is used in the derivation [11] of the Einstein's relation.

2.3 Disk partition functions for FQH droplets

To compute the partition function for the edge of a disk FQH sample we need some knowledge of the structure of the underlying CFT. The rational CFT for a FQH state always contains a $\widehat{u(1)}$ current algebra which is completely determined by the filling factor $\nu_H = n_H/d_H$. This current algebra always contributes a $c = 1$ stress-tensor to the Virasoro algebra due to the Sugawara contribution [12]. There is in general, a neutral Virasoro generator $T^{(0)}(z)$ as well, defined by $T(z) - T^{(c)}(z) = T^{(0)}(z)$ whose central charge must be positive.

The electron field operator naturally decomposes into a charged $\widehat{u(1)}$ part and a neutral component which must be a primary field of the neutral Virasoro algebra. From the electron CFT dimension $\Delta_{\text{el}} = \frac{d_H}{2n_H} + \Delta^{(0)}$ we see that its statistical angle $\theta/\pi = 2\Delta_{\text{el}} = 2\Delta^{(0)} + \frac{d_H}{n_H}$, which must be an odd integer, imposes certain conditions on the structure of the CFT. In particular, the electron field operator must have a non-trivial neutral component when $n_H > 1$, hence the neutral Virasoro algebra must be non-trivial, too. This also implies that the charged and neutral parts of the RCFT are not completely independent and therefore the partition function will not be simply a product of charged and neutral partition functions – instead there are pairing rules for the admissible combinations of charged and neutral characters.

3 Decomposable subalgebra and \mathbb{Z}_{n_H} grading

In this section we will consider in more detail the algebraic structure of the rational CFT corresponding to a general FQH state on a disk.

We start by noting that the $\widehat{u(1)}$ part² of the electron field operator, constructed as a $\widehat{u(1)}$ vertex exponent [15], with a charge parameter determined by the filling factor,

$$:\psi_{\text{el}}(z)\overline{\Psi^{(0)}}(z): \simeq :e^{-i\frac{1}{\sqrt{\nu_H}}\phi^{(c)}(z)}:, \quad (12)$$

of a chiral boson normalized by

$$\langle \phi^{(c)}(z)\phi^{(c)}(w) \rangle = -\ln(z-w), \quad (13)$$

certainly commutes with all neutral field operators. However, the vertex exponent (12) has in general a non-integer statistical angle $\theta/\pi = d_H/n_H$ and is not local for

² this part can be considered as the result of the fusion of the full electron operator with its neutral component $\Psi^{(0)}(z)$

$n_H > 1$. Therefore it does not belong to the chiral (super)algebra \mathcal{A} and cannot be used to decompose the latter.

The way out of this locality problem is to consider the n_H -th power of the vertex exponent (12)

$$:\exp\left(-i\frac{n_H}{\sqrt{V_H}}\phi^{(c)}(z)\right): = :\exp\left(-i\sqrt{n_H d_H}\phi^{(c)}(z)\right): \quad (14)$$

which still commutes with all neutral field operators but is local because its statistics is $\theta/\pi = n_H d_H$, so that it does belong to \mathcal{A} . It is worth stressing that the $\widehat{u(1)}$ vertex operator (14) together with all neutral generators of \mathcal{A} generates a *decomposable chiral subalgebra* \mathcal{A}_D of the original chiral superalgebra \mathcal{A}

$$\mathcal{A}_D = \widehat{u(1)}_m \otimes \mathcal{A}^{(0)} \subset \mathcal{A}. \quad (15)$$

We use the notation $\widehat{u(1)}_m$ to denote the rational extension [12, 15] of the $\widehat{u(1)}$ current algebra with the pair of vertex exponents $:e^{\pm i\sqrt{m}\phi}(z):$ with $m = n_H d_H$.

Because the decomposable subalgebra \mathcal{A}_D misses only the powers of the full electron operator ψ_{el}^s with $s = 0, \dots, n_H - 1$, the original superalgebra \mathcal{A} can be naturally represented as the following *direct sum decomposition*

$$\mathcal{A} = \bigoplus_{s=0}^{n_H-1} \psi_{\text{el}}^s \mathcal{A}_D. \quad (16)$$

Due to the orthogonality of the different powers of the electron field, following from the $\widehat{u(1)}$ charge conservation,

$$\langle \psi_{\text{el}}^s \mathcal{A}_D, \psi_{\text{el}}^{s'} \mathcal{A}_D \rangle = \langle \psi_{\text{el}}^s, \psi_{\text{el}}^{s'} \rangle \langle \mathcal{A}_D, \mathcal{A}_D \rangle = 0 \quad \text{if } s \neq s',$$

where $\langle \dots, \dots \rangle$ denotes the scalar product, it appears that the decomposition in Eq. (16) is in fact a \mathbb{Z}_{n_H} -graded direct sum decomposition.

The virtue of having a decomposable subalgebra is that it defines the following *dual algebra inclusion*

$$\mathcal{A}_D \subset \mathcal{A} \subset \mathcal{A}^* \subset \mathcal{A}_D^*, \quad (17)$$

which simplifies the construction of the representation spaces. It follows from Eq. (17) that all representations of \mathcal{A} are also representations of \mathcal{A}_D and at the same time that not all representations of \mathcal{A}_D are true representations of \mathcal{A} .

Given that the decomposable algebra (15) is simply a tensor product, its irreducible representations (IR) are labeled by pairs of quantum numbers (l, Λ) , where l is the electric charge of the bulk quasiparticles in such units that $Q_{\text{el}}(\text{bulk}) = l/d_H$, and Λ is the (total) neutral topological charge of the bulk quasiparticles. Then, it follows from Eq. (16) that all IRs of \mathcal{A} are direct sums of IRs of \mathcal{A}_D , corresponding to the orbit of the simple current's action, hence we shall be labeling the irreducible representations of \mathcal{A} by the same pair (l, Λ) , corresponding to the $s = 0$ component in Eq. (16).

As follows from Eq. (17), not all representations of \mathcal{A}_D are true representations of the original superalgebra \mathcal{A} . In order to identify the *physical* excitations, corresponding to the true representations of \mathcal{A} we will require that they are local with respect to the electron field. The locality principle implies that those IRs of \mathcal{A}_D which are local with respect to the electron are also IRs of \mathcal{A} . To formulate more precisely the locality requirement let us consider the decomposition of the electron field and an arbitrary excitation labeled by (l, Λ) into $\widehat{u(1)}$ and neutral parts

$$\begin{aligned} \text{electron:} \quad \psi_{\text{el}}(z) &= :e^{-i\frac{d_H}{\sqrt{n_H d_H}}\phi^{(c)}(z)}: \Psi_{\omega}^{(0)}(z) \\ \text{excitation:} \quad \psi_{l,\Lambda}(z) &= :e^{i\frac{l}{\sqrt{n_H d_H}}\phi^{(c)}(z)}: \Psi_{\Lambda}^{(0)}(z), \end{aligned}$$

where the $\widehat{u(1)}$ boson is normalized as in Eq. (13), the electric charge is related to the $\widehat{u(1)}$ label l by $Q_{\text{el}}(l) = l/d_H$, so that the electric charge label of the electron is $l = -d_H$, and ω denotes the (nontrivial) neutral topological charge of the electron.

Now, to identify the *physical* excitations within the extended dual algebra \mathcal{A}_D^* we require local operator product expansion (OPE) of the excitation with respect to the electron, i.e., we require the power of the coordinate distance $(z-w)$ in the short-distance OPE to be integer

$$\psi_{\text{el}}(z)\psi_{l,\Lambda}(w) \underset{z \rightarrow w}{\simeq} (z-w)^{-\frac{l}{n_H} + Q_{\omega}(\Lambda)} :e^{i\frac{l-d_H}{\sqrt{n_H d_H}}\phi^{(c)}(z)}: \Psi_{\omega*\Lambda}^{(0)}(w),$$

where $Q_{\omega}(\Lambda)$ is the (neutral) monodromy charge defined by the following combination of conformal dimensions $\Delta_{\Lambda'}$ of the neutral Virasoro IRs

$$Q_{\omega}(\Lambda) \equiv \Delta_{\omega*\Lambda} - \Delta_{\Lambda} - \Delta_{\omega} \bmod \mathbb{Z}, \quad \left(\Delta_{\omega} = \Delta^{(0)}\right). \quad (18)$$

Thus, the locality condition implies that the physical excitations (respectively, the true IRs of \mathcal{A}) must satisfy the following \mathbb{Z}_{n_H} *pairing rule* which selects the admissible pairs (l, Λ) of charged and neutral quantum numbers

$$n_H Q_{\omega}(\Lambda) \equiv l \bmod n_H. \quad (19)$$

The representation spaces of $\mathcal{A}_D = \widehat{u(1)}_m \otimes \mathcal{A}^{(0)}$ labeled by the pairs (l, Λ) which obey the PR (19) (that guarantees these pairs are true representations of the original algebra \mathcal{A}) are naturally tensor products of the representation spaces $\mathcal{H}_l^{(c)}$ for the $\widehat{u(1)}$ current algebra and those, $\mathcal{H}_{\Lambda}^{(0)}$, for the neutral Virasoro algebra, i.e.

$$\mathcal{H}_{l,\Lambda}^D = \mathcal{H}_l^{(c)} \otimes \mathcal{H}_{\Lambda}^{(0)}, \quad (20)$$

which explains why we looked for a decomposable subalgebra.

The representation spaces $\mathcal{H}_{l,\Lambda}^{\mathcal{A}}$ for the original algebra \mathcal{A} can be obtained by the action of \mathcal{A} over the lowest-weight state $|l, \Lambda\rangle$. Because of the decomposition (16) this space has a natural direct sum decomposition into representation space $\mathcal{H}_{l,\Lambda}^D$ for

the decomposable subalgebra

$$\mathcal{H}_{l,\Lambda}^{\mathcal{A}} = \mathcal{A}|l, \Lambda\rangle = \bigoplus_{s=0}^{n_H-1} \psi_{\text{el}}^s \mathcal{A}^D |l, \Lambda\rangle = \bigoplus_{s=0}^{n_H-1} \mathcal{J}^s \mathcal{H}_{l,\Lambda}^D,$$

where $\mathcal{J} \simeq \psi_{\text{el}}^*(0)$ is the simple current [12] representing the action of the electron field operator over the lowest-weight states, i.e.

$$\mathcal{J}|l, \Lambda\rangle = |l + d_H, \omega * \Lambda\rangle,$$

which means that the simple current \mathcal{J} acts on lowest-weight states by fusion - the $\widehat{u(1)}$ charge is simply shifted by the electric charge of the electron, while the neutral Virasoro topological charges are fused with that of the electron.

Taking into account Eq. (20) we finally obtain the representation space for \mathcal{A}

$$\mathcal{H}_{l,\Lambda}^{\mathcal{A}} = \bigoplus_{s=0}^{n_H-1} \mathcal{J}^s \left(\mathcal{H}_l^{(c)} \otimes \mathcal{H}_\Lambda^{(0)} \right) = \bigoplus_{s=0}^{n_H-1} \mathcal{H}_{l+sd_H}^{(c)} \otimes \mathcal{H}_{\omega^s * \Lambda}^{(0)}. \quad (21)$$

The benefit of this representation of the Hilbert space for a general FQH disk is that its $\widehat{u(1)}$ part $\mathcal{H}_l^{(c)}$, which is the edge-states' space of the Luttinger liquid, is completely determined by the filling factor ν_H and the neutral part $\mathcal{H}_\Lambda^{(0)}$ is what distinguishes between FQH states with the same filling factor but different universality classes.

4 The RCFT partition function for a general FQH disk

Now that we know the general structure of the Hilbert space for an arbitrary FQH disk state we can obtain the corresponding structure of the partition function by plugging Eq. (21) into Eq. (3). Notice however, that the $\widehat{u(1)}_m$ representation spaces $\mathcal{H}_l^{(c)}$ entering Eq. (21) correspond to $m = n_H d_H$ and therefore the electric charge operator Q could be represented in terms of $\widehat{u(1)}_m$ number operator $N = J_0 / \sqrt{m}$, i.e.

$$Q = \sqrt{\frac{n_H}{d_H}} J_0 = \sqrt{\frac{n_H}{d_H}} \sqrt{n_H d_H} N = n_H N. \quad (22)$$

Therefore, using the properties of the trace as well as the structure of the Hilbert space (21), we obtain the main result—the partition function for a general FQH disk can be represented as a sum of n_H products of $\widehat{u(1)}$ and neutral partition functions

$$Z_{l,\Lambda}(\tau, \zeta) = \sum_{s=0}^{n_H-1} K_{l+sd_H}(\tau, n_H \zeta; n_H d_H) \text{ch}_{\omega^s * \Lambda}(\tau), \quad (23)$$

where the $\widehat{u(1)}$ partition functions $K_{l+sd_H}(\tau, n_H \zeta; n_H d_H)$ are expressed as Luttinger liquid partition functions for $m = n_H d_H$ in the notation of [15]

$$K_l(\tau, \zeta; m) = \frac{\text{CZ}(\tau, \zeta)}{\eta(\tau)} \sum_{n=-\infty}^{\infty} q^{\frac{m}{2}(n+\frac{l}{m})^2} e^{2\pi i \zeta(n+\frac{l}{m})}. \quad (24)$$

The absolute temperature and the Boltzmann factor $e^{-\beta}$ are related to the modular parameter τ

$$q = e^{-\beta \Delta \varepsilon} = e^{2\pi i \tau}, \quad \Delta \varepsilon = \hbar \frac{2\pi v_F}{L}, \quad (25)$$

where $\Delta \varepsilon$ is the non-interacting energy spacing, v_F is the Fermi velocity on the edge and L is the circumference of the disk. The Dedekind function η and Cappelli–Zemba factors [16] entering Eq. (24) are explicitly given by

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad \text{CZ}(\tau, \zeta) = e^{-\pi v_H \frac{(\text{Im} \zeta)^2}{\text{Im} \tau}}.$$

It is worth stressing that the $\widehat{u(1)}$ partition functions (24) are completely explicit and totally determined by the filling factor's numerator n_H and denominator d_H . The extra n_H in front of ζ in the Luttinger-liquid partition function $K_{l+sd_H}(\tau, n_H \zeta; n_H d_H)$ appears due to the relation (22).

The neutral partition functions, which are known mathematically as the characters of the representations $\mathcal{H}_\Lambda^{(0)}$ of the neutral Virasoro algebra with central charge $c - 1$, are defined as usual as the trace over the representation space [12]

$$\text{ch}_\Lambda(\tau) = \text{tr}_{\mathcal{H}_\Lambda^{(0)}} q^{L_0^{(0)} - \frac{c-1}{24}}.$$

The neutral topological charge of the electron is denoted by ω and $\omega * \Lambda$ in Eq. (23) denotes the fusion of the topological charges of the electron and the bulk quasiparticles. Unlike the charged-part partition functions the neutral ones are not completely determined by the filling factor, though their structure is almost fixed by the neutral weights ω, Λ and their fusion rules, thus representing more subtle topological properties of the FQH universality class. Fortunately, for most of the FQH universality classes these functions are explicitly known.

5 Application: Coulomb blockade in the \mathbb{Z}_3 Read–Rezayi state

The structure of the partition function (23), in which the $\widehat{u(1)}$ part is explicitly separated, is very convenient for the computation of the CB peaks for a FQH island at finite temperature since the variation of the AB flux ϕ changes only the $\widehat{u(1)}$ partition functions (24) because of Eq. (5). Consider, for example a CB island in which the FQH state is the \mathbb{Z}_3 Read–Rezayi (parafermion) state [17, 18], characterized by

$n_H = 3$, $d_H = 5$, i.e. $\nu_H = 3/5$. The decomposable chiral subalgebra is $\widehat{u(1)}_{15} \times \mathcal{W}_3$, where \mathcal{W}_3 is the \mathbb{Z}_3 parafermion algebra of Fateev-Zamolodchikov [19]. The neutral part of the electron operator has a topological charge $\omega = \psi_1$, $\omega^2 = \psi_2$ given by the parafermion currents. As a simple illustration of the entire procedure let us consider the case when there are no quasiholes in the bulk, which corresponds to $l = 0$, $\Lambda = 0$. The partition function (23) takes the form

$$Z_{0,0}(\tau, \zeta) = K_0(\tau; 3\zeta; 15) \text{ch}_{00}(\tau) + K_5(\tau; 3\zeta; 15) \text{ch}_{01}(\tau) + K_{-5}(\tau; 3\zeta; 15) \text{ch}_{02}(\tau)$$

where the K functions are defined in Eq. (24), the Boltzmann factor q is defined in Eq. (25) and the neutral partition functions are defined by

$$\text{ch}_{0,l}(\tau) = q^{-\frac{1}{30}} \sum_{n_1, n_2 \geq 0}^{(l)} \frac{q^{\frac{2}{3}(n_1^2 + n_2 n_1 + n_2^2)}}{(q)_{n_1} (q)_{n_2}}, \quad (q)_n = \prod_{j=1}^n (1 - q^j).$$

and the sum $\sum^{(l)}$ is restricted by the condition $n_1 + 2n_2 = l \bmod 3$. Introducing AB flux as in Eq. (5) and plugging the partition function with flux into Eq. (11) we calculate numerically the conductance of the CB island at temperature $T = 0.5T_0$ as the flux is varied, see Fig. 2. Under the assumption that the neutral and charged

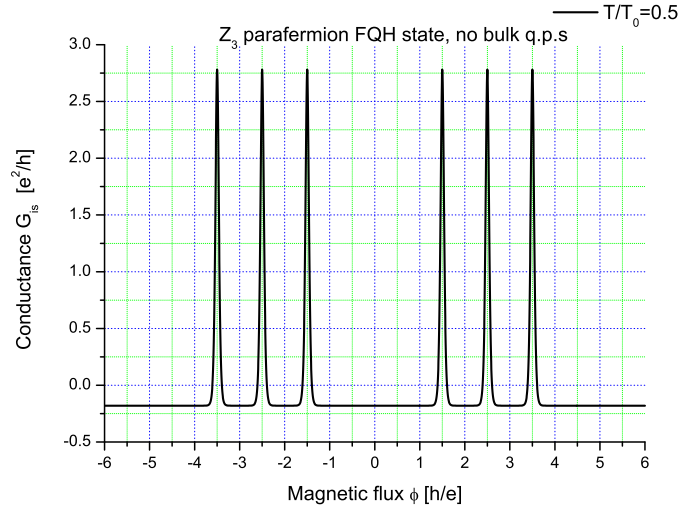


Fig. 2 Coulomb blockade peaks of the conductance, appearing when AB flux is varied, for the \mathbb{Z}_3 parafermion FQH island without non-trivial quasiparticles in the bulk at temperature $T/T_0 = 0.5$.

modes propagate with the same Fermi velocity we see that the CB peaks are clustered in bunches of three, separated by flux period $\Delta\phi_1 = 1$ inside the bunch, and

separated by a flux period $\Delta\phi_2 = 3$ between the bunches, which is in agreement with the previous results at zero temperature [7, 9, 10].

Most of the characteristics of the CB peaks, such as the height, the width and the periods, can be derived asymptotically at very low temperatures [5].

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